Moment Distributions Around Holes in Symmetric Composite Laminates Subjected to Bending Moments

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An analytical investigation of the effects of holes on the moment distribution of symmetric composite laminates subjected to bending moments is described. A general, closed-form solution for the moment distribution of an infinite anisotropic plate is derived, and this solution is used to determine stress distributions both on the hole boundary and throughout the plate. Results are presented for several composite laminates that have holes and are subjected to either pure bending or cylindrical bending. Laminates with a circular hole or with an elliptical hole are studied. Laminate moment distributions are discussed, and ply stresses are described.

Introduction

STRUCTURALLY efficient designs for composite structures require a thorough understanding of the response of composite laminates to fundamental loading conditions. Many researchers have studied the response of composite laminates subjected to inplane loadings. In general, these studies focused on the response of tension-, compression-, or shear-loaded laminates with and without holes. Bending and twisting moments are also fundamental loading conditions. The response of homogenous thin plates subjected to these moments has been studied.[1,2] Naghdi[3] studied the effect of elliptical holes on the bending of thick isotropic plates. Pickett[4] and Lo and Leissa[5] investigated isotropic plates with central holes subjected to lateral loads. Moriya[6] conducted a finite-element analysis to determine stresses around both holes and cracks in isotropic plates subjected to bending moments. Delale et al.[7] considered the axisymmetric bending of a plate made of two bonded dissimilar isotropic materials and containing a central hole.

Analyses to determine the effects of holes on the stress field for composite laminates subjected to various loading conditions include both approximate and closed-form solutions. Approximate solutions are usually based on specific functions that represent the problem unknowns and minimize the error between these functions and known results at discrete points. These solution procedures can be computationally inefficient. Closed-form solutions determine the unknowns exactly as continuous functions and these solutions are well-suited for parametric studies because of their computational efficiency. Closed-form solutions have been obtained for determining the effects of a hole on the stress field for composite laminates subjected to inplane loads.[8-11] Also, closed-form solutions have been obtained for the stress distribution around a pin-loaded hole in composite laminates subjected to inplane loads.[12-14] Some investigators have obtained solutions for the more complex problem of multiple holes in composite laminates subjected to inplane loads.[15,16] The response of composite plates with a hole and subjected to bending or twisting moments has received limited attention. Moment distributions on the hole boundary only for homogenous orthotropic plates subjected to bending moments are given in Ref. 1. The solutions technique outlined in Ref. 1 has not been used to determine moment distributions throughout the plate and has not been extended to composite laminates. A general, closed-form solution is needed to determine the effects of holes on the moment distribution for an entire composite laminate subjected to bending moments.

The objective of the present paper is to determine analytically the effects of holes on the moment distribution of symmetric composite laminates subjected to bending moments. The governing equations are derived for an infinite anisotropic plate with a hole. A linear, closed-form solution to these equations is presented for a plate subjected to far-field bending moments. The solution is based on the method of conformal mapping developed by Muskhelishvili[17] for the two-dimensional theory of elasticity and is computationally efficient. A similar technique has been used in Ref. 2 to determine the effects of holes on the stress field in homogenous plates subjected to bending moments. As will be explained later, the results in Ref. 2 are inaccurate for orthotropic and anisotropic plates. The present solution determines the results for isotropic, orthotropic, and anisotropic plates at the hole boundary and throughout the plate. The accuracy of this solution is verified by comparing the moment distributions on the hole boundary for both an isotropic plate and a unidirectional laminated plate with similar moment distributions obtained using Ref. 1. This present solution is used to determine the moment distributions around a hole for several symmetric composite laminates subjected to cylindrical bending or pure bending. Ply stresses for a quasi-isotropic laminate having a circular hole and subjected to pure bending are discussed.

Theory

This section outlines the governing equations for an infinite anisotropic plate that has a hole and is subjected to far-field bending moments. A solution to these equations is described that determines the unknown functions explicitly using far-field boundary conditions.

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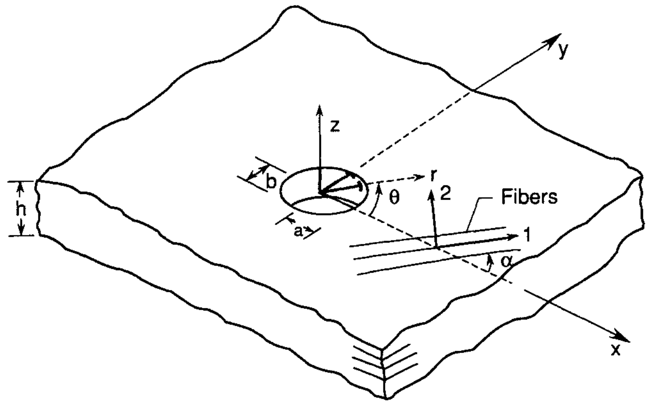

Fig. 1 Geometry and coordinate systems for a composite laminate.

The geometry and coordinate systems used in this study are shown in Fig. 1. The plate has thickness h. The semiminor and semimajor axes of the hole have dimensions a and b, respectively. The a dimension is measured along the x axis, and the b dimension is measured along the y axis. The fiber orientation angle α for a ply is also measured with respect to the x axis. The r,θ coordinate system is also used to describe the results.

The governing equations are derived using the classical Kirchoff-Love assumptions for plates. These assumptions lead to the expressions

$$u(x,y,z) = -z\frac{\partial w(x,y)}{\partial x} \tag{1a}$$

$$v(x,y,z) = -z\frac{\partial w(x,y)}{\partial y} \tag{1b}$$

$$w(x,y,z) = w(x,y) \tag{1c}$$

where u, v, and w are displacements in the x, y, and z directions, respectively. The inplane strains for the plate are

$$\epsilon_x = -z\frac{\partial^2 w}{\partial x^2} \tag{2a}$$

$$\epsilon_y = -z\frac{\partial^2 w}{\partial y^2} \tag{2b}$$

$$\gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y} \tag{2c}$$

and the moment resultants for the symmetric laminated plate are

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} -\dfrac{\partial^2 w}{\partial x^2} \\ -\dfrac{\partial^2 w}{\partial y^2} \\ -2\dfrac{\partial^2 w}{\partial x \partial y} \end{Bmatrix} \tag{3}$$

where D_{mn} are the plate bending stiffnesses. The governing differential equation for the plate is expressed as

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} = 0 \tag{4}$$

The assumed form $w = Ae^{\xi(x+sy)}$, where A and ξ are constants, is substituted into Eq. (4) to obtain the characteristic equation

$$D_{22}s^4 + 4D_{26}s^3 + 2(D_{12} + 2D_{66})s^2 + 4D_{16}s + D_{11} = 0 \tag{5}$$

Lekhnitskii[1] has shown that Eq. (5) has no real roots. In general, these roots are not equal and can be represented as

$$s_1 = \alpha_1 + i\beta_1, \qquad s_2 = \alpha_2 + i\beta_2 \tag{6a}$$

$$s_3 = \bar{s}_1, \qquad s_4 = \bar{s}_2\ (\beta_1 > 0,\ \beta_2 > 0) \tag{6b}$$

where the quantities s_1 and s_2 are complex parameters, and bars over complex parameters (e.g., $\bar{s}_1$ and $\bar{s}_2$) indicate complex conjugates.

The general expression for w that satisfies Eq. (4) for an anisotropic plate with $s_1 \neq s_2$ can be written as

$$w(x,y) = 2Re\,[F_1(z_1) + F_2(z_2)] \tag{7}$$

where $F_1(z_1)$ and $F_2(z_2)$ are arbitrary analytic functions having arguments

$$z_1 = x + s_1 y, \qquad z_2 = x + s_2 y \tag{8}$$

The solution procedure outlined here is only for unequal complex parameters ($s_1 \neq s_2$). Equation (7) is substituted into Eq. (3) to obtain

$$M_x = -2Re\,[p_1\phi'(z_1) + q_1\psi'(z_2)] \tag{9a}$$

$$M_y = -2Re\,[p_2\phi'(z_1) + q_2\psi'(z_2)] \tag{9b}$$

$$M_{xy} = -2Re\,[p_3\phi'(z_1) + q_3\psi'(z_2)] \tag{9c}$$

$$Q_x = -2Re\,[s_1 p_4\phi''(z_1) + s_2 q_4\psi''(z_2)] \tag{9d}$$

$$Q_y = 2Re\,[p_4\phi''(z_1) + q_4\psi''(z_2)] \tag{9e}$$

where Q_x and Q_y are shear stress resultants and

$$\phi(z_1) = \frac{dF_1}{dz_1}, \qquad \psi(z_2) = \frac{dF_2}{dz_2}$$

$$\phi'(z_1) = \frac{d^2F_1}{dz_1^2}, \qquad \psi'(z_2) = \frac{d^2F_2}{dz_2^2}$$

$$\phi''(z_1) = \frac{d^3F_1}{dz_1^3}, \qquad \psi''(z_2) = \frac{d^3F_2}{dz_2^3}$$

where

$$p_1 = D_{11} + D_{12}s_1^2 + 2s_1 D_{16}$$

$$p_2 = D_{12} + D_{22}s_1^2 + 2s_1 D_{26}$$

$$p_3 = D_{16} + D_{26}s_1^2 + 2s_1 D_{66}$$

$$p_4 = \frac{D_{11}}{s_1} + 3D_{16} + s_1(D_{12} + 2D_{66}) + D_{26}s_1^2$$

$$q_1 = D_{11} + D_{12}s_2^2 + 2s_2 D_{16}$$

$$q_2 = D_{12} + D_{22}s_2^2 + 2s_2 D_{26}$$

$$q_3 = D_{16} + D_{26}s_2^2 + 2s_2 D_{66}$$

$$q_4 = \frac{D_{11}}{s_2} + 3D_{16} + s_2(D_{12} + 2D_{66}) + D_{26}s_2^2$$

The functions $\phi(z_1)$ and $\psi(z_2)$ for an infinite plate with a hole have the form

$$\phi(z_1) = (B^* + iC^*)\,z_1 + \phi_0(z_1) \tag{10a}$$

$$\psi(z_2) = (B'^* + iC'^*)\,z_2 + \psi_0(z_2) \tag{10b}$$

where $\phi_0(z_1)$ and $\psi_0(z_2)$ are analytic functions outside the hole.

The constant coefficients B^*, B'^*, C^*, and C'^* are determined from the far-field boundary conditions. At infinity, the functions $\phi_0(z_1)$ and $\psi_0(z_2)$ are zero. Substituting Eqs. (10) into Eqs. (9) for moments at infinity results in

$$M_x^\infty = -2Re\,[p_1(B^* + iC^*) + q_1(B'^* + iC'^*)] \tag{11a}$$

$$M_y^\infty = -2Re\,[p_2(B^* + iC^*) + q_2(B'^* + iC'^*)] \tag{11b}$$

$$M_{xy}^\infty = -2Re\,[p_3(B^* + iC^*) + q_3(B'^* + iC'^*)] \tag{11c}$$

Equations (11) are only three equations involving four coefficients. The solution in Ref. 2 assumes that one of the constant coefficients B^*, C^*, B'^*, and C'^* can be arbitrarily chosen, and selects $C^* = 0$. This solution yields correct results for isotropic plates but yields incorrect results for orthotropic and anisotropic plates. In the present formulation, another far-field condition is imposed to yield a fourth equation for determining the four constant coefficients. The functions $\phi''(z_1)$ and $\psi''(z_2)$ are zero for large z_1 and z_2 causing the far-field shear stress resultants Q_x^∞ and Q_y^∞ to equal zero. Consequently, the fourth equation is

$$\begin{aligned} P_z^* &= -2Re\,[p_4\phi'(z_1) + q_4\psi'(z_2)] \\ &= -2Re\,[p_4(B^* + iC^*) + q_4(B'^* + iC'^*)] = 0 \end{aligned} \tag{12}$$

where P_z^* is force resultant in the z direction. The set of equations given by Eqs. (11) and (12) are sufficient for determining all four constant coefficients B^*, B'^*, C^*, and C'^*.

The final expressions for the functions $\phi_0(z_1)$ and $\psi_0(z_2)$ are

$$\phi_0(z_1) = \frac{N_1(a - is_1b)}{z_1 + [z_1^2 - (a^2 + s_1^2b^2)]^{1/2}} \tag{13a}$$

$$\psi_0(z_2) = \frac{N_2(a - is_2b)}{z_2 + [z_2^2 - (a^2 + s_2^2b^2)]^{1/2}} \tag{13b}$$

where the complex coefficients N_1 and N_2 are of the form

$$\begin{aligned} N_1 = \frac{s_1}{2(p_1q_2s_2 - p_2q_1s_1)} \Bigg\{ &(B^* + iC^*)\left(p_2q_1 - \frac{p_1q_2s_2}{s_1}\right) \\ &\times (a + is_1b) + (B^* - iC^*)\left(\bar{p}_2q_1 - \frac{\bar{p}_1q_2s_2}{\bar{s}_1}\right)(a + i\bar{s}_1b) \\ &+ (B'^* - i_C'^*)\left(q_1\bar{q}_2 - \frac{q_2\bar{q}_1s_2}{\bar{s}_2}\right)(a + i\bar{s}_2b)\Bigg\} \end{aligned} \tag{14a}$$

$$\begin{aligned} N_2 = \frac{-s_2}{2(p_1q_2s_2 - p_2q_1s_1)} \Bigg\{ &(B^* - iC^*)\left(p_1p_2 - \frac{p_2\bar{p}_1s_1}{\bar{s}_1}\right) \\ &\times (a + i\bar{s}_1b) + (B'^* + iC'^*)\left(p_1q_2 - \frac{p_2q_1s_1}{s_2}\right)(a + is_2b) \\ &+ (B'^* - iC'^*)\left(p_1\bar{q}_2 - \frac{p_2\bar{q}_1s_1}{\bar{s}_2}\right)(a + i\bar{s}_2b)\Bigg\} \end{aligned} \tag{14b}$$

and these expressions are obtained using the procedure described in Ref. 2.

Substituting the expressions for $\phi_0(z_1)$ and $\psi_0(z_2)$ from Eqs. (13) into Eqs. (10) provides the expressions for the functions $\phi(z_1)$ and $\psi(z_2)$. Derivatives of $\phi(z_1)$ and $\psi(z_2)$ are obtained from Eqs. (10) and are expressed as

$$\phi'(z_1) = (B^* + iC^*) + \phi_0'(z_1) \tag{15a}$$

$$\psi'(z_2) = (B'^* + iC'^*) + \psi_0'(z_2) \tag{15b}$$

Table 1 Material properties

	Isotropic	Graphite-epoxy[a]
Longitudinal Young's modulus E_1, GPa	72.4	159.00
Transverse Young's modulus E_2, GPa	72.4	10.00
Shear modulus G_{12}, GPa	27.8	5.90
Major Poisson's ratio, ν_{12}	0.3	0.31

[a]Nominal ply thickness is 0.15 mm.

Table 2 Moments on the boundary of circular hole for an infinite plate subjected to applied far-field moments

Angle θ, deg	M_θ, N-m/m Ref. 1	M_θ, N-m/m Present	$M_{r\theta}$, N-m/m Ref. 1	$M_{r\theta}$, N-m/m Present
a. Isotropic plate				
Cylindrical Bending: $M_x^\infty = 1.0$ N-m/m $M_y^\infty = M_{xy}^\infty = 0$				
0	0.21261	0.21212	0.00000	0.00000
30	0.60675	0.60106	−0.52485	−0.52487
60	1.39356	1.39334	−0.52485	−0.52486
90	1.78783	1.78788	0.00000	0.00000
Pure Bending: $M_x^\infty = M_y^\infty = 1.0$ N-m/m $M_{xy}^\infty = 0$				
0	1.99926	1.99999	0.00000	0.00000
30	1.99951	2.00000	0.00000	0.00000
60	1.99951	2.00000	0.00000	0.00000
90	1.99926	2.00000	0.00000	0.00000
b. Unidirectional plate				
Cylindrical Bending: $M_x^\infty = 1.0$ N-m/m $M_y^\infty = M_{xy}^\infty = 0$				
0	0.15069	0.15069	0.00000	0.00000
30	0.20724	0.20724	−0.07526	−0.07526
60	1.10160	1.10160	−0.50155	−0.50155
90	3.08159	3.08159	0.00000	0.00000
Pure Bending: $M_x^\infty = M_y^\infty = 1.0$ N-m/m $M_{xy}^\infty = 0$				
0	1.67267	1.67267	0.00000	0.00000
30	1.39294	1.39294	0.62633	0.62633
60	1.01812	1.01812	0.48208	0.48208
90	5.47811	5.47811	0.00000	0.00000

The expressions for the moments obtained by the substitution of Eqs. (15) into Eqs. (9) are

$$M_x = M_x^\infty - 2Re\,[p_1\phi_0'(z_1) + q_1\psi_0'(z_2)] \tag{16a}$$

$$M_y = M_y^\infty - 2Re\,[p_2\phi_0'(z_1) + q_2\psi_0'(z_2)] \tag{16b}$$

$$M_{xy} = M_{xy}^\infty - 2Re\,[p_3\phi_0'(z_1) + q_3\psi_0'(z_2)] \tag{16c}$$

The expressions for the derivatives $\phi_0'(z_1)$ and $\psi_0'(z_2)$ as obtained from Eqs. (13) are

$$\phi_0'(z_1) = \frac{N_1}{a + is_1b}\left[1 - \frac{z_1}{[z_1^2 - (a^2 + s_1^2b^2)]^{1/2}}\right] \tag{17a}$$

$$\psi_0'(z_2) = \frac{N_2}{a + is_2b}\left[1 - \frac{z^2}{[z_2^2 - (a^2 + s_2^2b^2)]^{1/2}}\right] \tag{17b}$$

The present solution provides the complete moment distribution for an infinite anisotropic plate that has an elliptical or a circular hole and is subjected to far-field bending moments. The moment distribution in polar coordinates can be obtained by using coordinate transformation.

Results and Discussion

This section describes analytical results that were obtained using the solution developed during this study. Results obtained using the solution presented herein are compared with classical results obtained using the solution in Ref. 1. This improved solution is used to obtain results for several composite laminates subjected either to cylindrical bending or pure bending. Ply results for a quasiisotropic laminate that has a circular hole and is subjected to pure bending are discussed. Material properties used for this study are presented in Table 1. The isotropic material properties are typical for aluminum, and the graphite-epoxy material properties are typical for an advanced, damage-tolerant graphite-epoxy material system.

Comparison with Previous Work

Moment distribution results are presented in Table 2. Results obtained using Ref. 1 and results obtained using the present solution are given in the table to illustrate the accuracy of the improved solution. The distribution of the moments on the hole boundary are calculated for a far-field cylindrical bending loading (i.e., $M_x^\infty = 1.0$ N-m/m, $M_y^\infty = M_{xy}^\infty = 0$) and for a far-field pure bending loading (i.e., $M_x^\infty = M_y^\infty = 1.0$ N-m/m, $M_{xy}^\infty = 0$). Results for an isotropic plate are presented in Table 2a, and results for a unidirectional laminated plate, ($\alpha = 0$ deg, see Fig. 1) are presented in Table 2b. For the isotropic plate, $s_1 = s_2 = i$. In the present calculations, the parameters s_1 and s_2 are intentionally made slightly unequal to avoid numerical singularities.[18] The moments calculated using the present solution differ from the corresponding moments calculated using Ref. 1 by less than 1% for the isotropic plate. The moments calculated using the two solutions are exactly the same for the unidirectional plate.

Laminate Results

The present solution is used to obtain moment distribution results for composite laminates. The laminates studied herein are designated as follows:

Laminate	Stacking sequence
A	$[0]_{48t}$
B	$[+45/0/-45/90]_{6s}$
C	$[+45/0/-45/0/+45/0/-45/90]_{3s}$
D	$[\pm 45/\mp 45]_{6s}$

All laminates have 48 plies. Laminate A is a highly orthotropic unidirectional laminate. Laminates B and C are quasiisotropic and orthotropic laminates, respectively, and are typical of laminates used for composite structures. Laminate D is a generally orthotropic laminate that is used for shear-dominated composite structures.

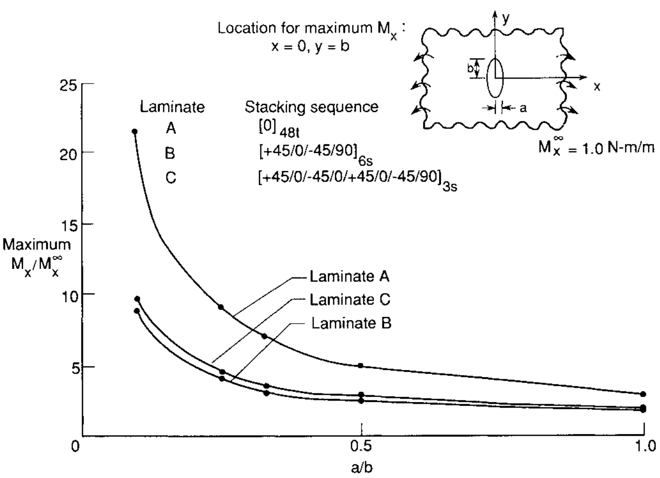

Fig. 2 Maximum normalized moments for composite laminates that have a hole and are subjected to cylindrical bending.

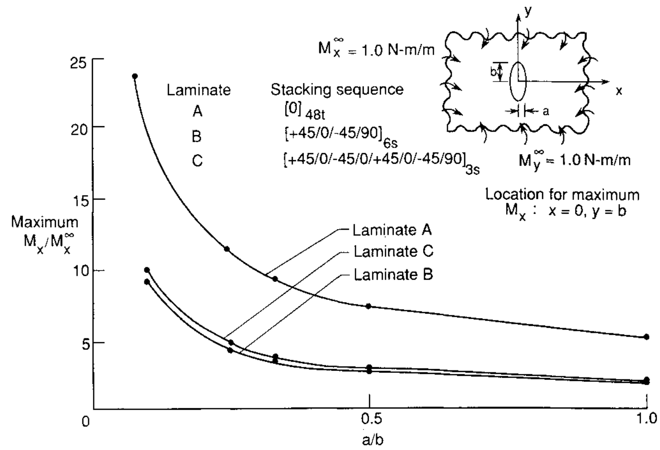

Fig. 3 Maximum normalized moments for composite laminates that have a hole and are subjected to pure bending.

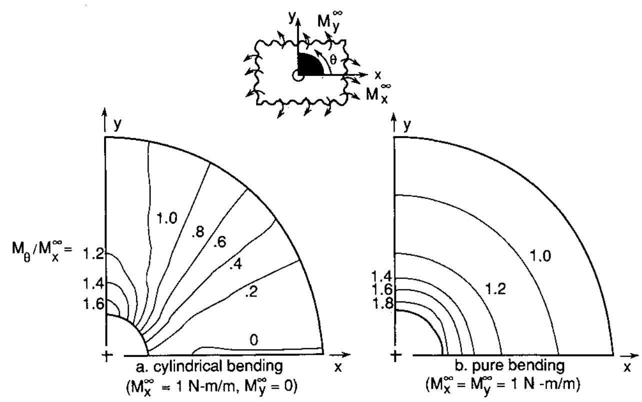

Fig. 4 Normalized moments for a $[+45/0/-45/90]_{6s}$ laminate (laminate B).

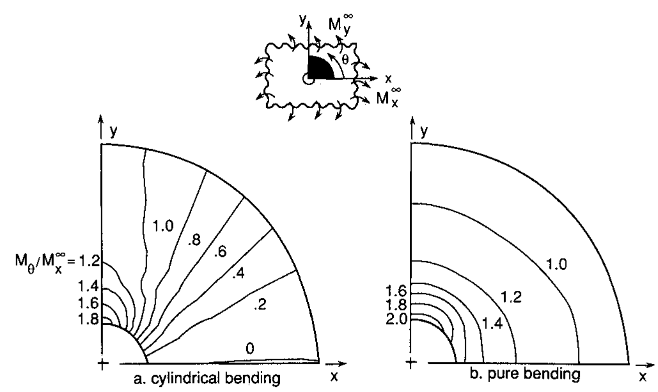

Fig. 5 Normalized moments for a $[+45/0/-45/0/+45/0/-45/90]_{3s}$ laminate (laminate C).

Results for the maximum normalized moment as a function of hole geometry are presented in Figs. 2 and 3 for laminates A, B, and C subjected to cylindrical bending (Fig. 2) or to pure bending (Fig. 3). The hole geometry is described using the ratio of the semiminor axis to the semimajor axis a/b, and this ratio is plotted along the abscissa. The maximum moment resultant M_x for these laminates subjected to these loads occurs at $x = 0$, $y = b$ (see Fig. 1), and this moment resultant is normalized by the corresponding far-field moment resultant $M_x^\infty (x = y = \infty)$ and is plotted along the ordinate. The maximum normalized moment resultants for laminate D do not occur at the same location as the results for laminates A, B, and C. The results for laminate D are not included in Figs. 2 and 3, and the moment distribution results for laminate D are discussed subsequently. The results in Figs. 2 and 3 show that the maximum normalized moment resultant for each laminate are approximately the same for cylindrical bending loads and for pure bending loads. The maximum normalized moment resultant for a given a/b value increases with an increasing level of laminate orthotropy for these laminates. The maximum normalized moment resultant for laminate B or C with a circular hole and subjected to these loadings is approximately 2.

Contour plots of the normalized M_θ/M_x^∞ for laminates B, C, and D that have a circular hole and are subjected to either cylindrical bending or pure bending are shown in Figs. 4–6, respectively. Results for laminates subjected to cylindrical bending are shown in Figs. 4a, 5a, and 6a, and results for laminates subjected to pure bending are shown in Figs. 4b, 5b, and 6b. The results in Figs. 4 and 5 indicate that the moment distributions for a given loading condition are similar for laminates B and C. The moments near the hole for the orthotropic laminate C are slightly higher than the moments near the hole for the quasiisotropic laminate B. The results in Fig. 6 for laminate D indicate a skewing of the moment distribution when compared to the results for laminates B and C. This skewing may be caused by the D_{66} bending stiffness of laminate D being significantly greater than the D_{66} bending stiffness for laminates B and C. The maximum moment occurs on the boundary of the hole and at $\theta = 60$ deg and at $\theta = 45$ deg for the cylindrical bending loading and for the pure bending loading, respectively.

Ply Stresses

Ply results are obtained using the laminate moments and classical laminated plate theory.[19] Equations (2) and (3) are used to calculate the strains for a ply. Ply stresses are calculated using these strains and a constitutive relation. The ply stresses are presented for a coordinate system that corresponds to directions parallel and perpendicular to the fibers in the plane of the ply (see Fig. 1). The direction parallel to the fiber is the 1 direction, and the direction perpendicular to the fibers is the 2 direction. Maximum normalized stresses at the hole boundary for the outermost plies of a $[+45/0/-45/90]_{6s}$ laminate are shown in Fig. 7. The laminate has a circular hole and is subjected to pure bending. The ply stresses are normalized by the maximum axial far-field stress where

$$\sigma_x^\infty = \frac{6M_x^\infty}{h^2} \tag{18}$$

The results indicate that for this loading and laminate type, significant stresses develop in the fiber direction for both the outermost 0-deg and 90-deg plies. These results suggest that laminate failure may initiate due to either a tensile or compressive failure in the fiber direction for these outermost plies. The type of failure will depend on the tensile and compressive strengths.

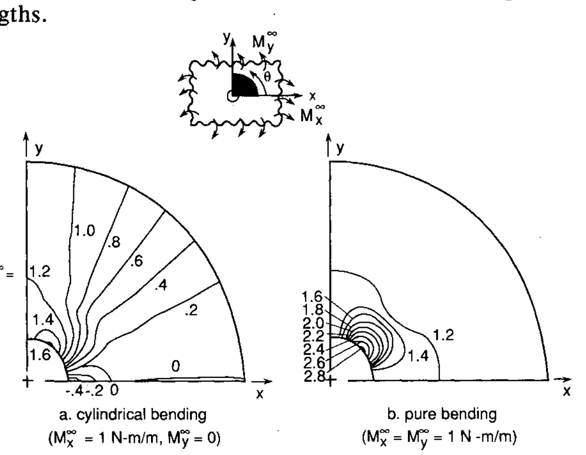

Fig. 6 Normalized moments for a $[\pm 45/\mp 45]_{6s}$ laminate (laminate D).

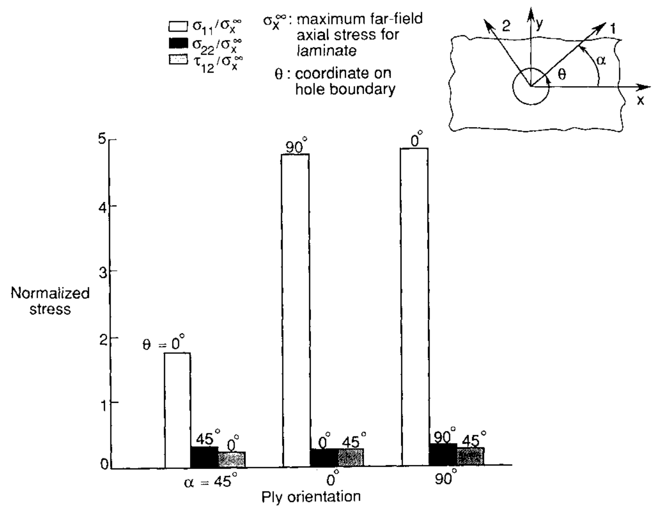

Fig. 7 Maximum normalized stresses for the outermost plies of a $[+45/0/-45/90]_{6s}$ laminate (laminate B) that has a circular hole and is subjected to pure bending.

Concluding Remarks

This paper describes analytical results for the effects of holes on the moment field of symmetric composite laminates subjected to bending moments. A general, closed-form solution for an infinite anisotropic plate has been derived, and this solution has been used to determine stress distributions, both on the hole boundary and throughout the laminate. The solution utilizes the method of conformal mapping and complex variables and determines the unknown stress functions explicitly using far-field boundary conditions.

The solution presented in this paper is used to obtain results for several symmetric composite laminates. Normalized moment resultants for laminates with holes and subjected to either pure bending or to cylindrical bending are described. The normalized moment resultant for the laminates considered in this study that are subjected to either pure bending or cylindrical bending are approximately the same. The normalized moment resultants for the quasiisotropic and the orthotropic laminates with a circular hole considered in this study are approximately equal to two. Ply stresses for a quasiisotropic laminate that has a circular hole and is subjected to pure bending are discussed. The stresses in the fiber direction are significant for the outermost 0-deg and 90-deg plies, and these stresses may initiate tensile or compressive failure in the fiber direction. The type of failure will be a function of the tensile and compressive strengths.

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